4^{th} SEM B.COM /BBA

CALICUT UNIVERSITY

BASIC NUMERICAL METHODS

2019 ADMISSION

Prepared by

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BCM3A11 BASIC NUMERICAL METHODS

Lecture Hours per week: 5, Credits: 4

Internal: 20, External: 80, Examination 2.5 Hours

Objectives:

To enable the students to acquire knowledge of numerical equations, matrices progressions, financial mathematics and descriptive statistics. At the end of this course, the students will be able to understand, numerical equations, matrix, progression, financial mathematics, descriptive statistics and their applications.

Module I

Numerical expressions and Equations: Simultaneous linear equations (up to three variables), Quadratic equations in one variable-factorization and quadratic formula *(10 Hours, 10 marks)*

Module II

Matrices: introduction - type of matrices – trace and transpose and determinants - matrix operations – adjoint and inverse –rank- solving equations by matrices: Cramer‘s Rule (not more than three variables). *(15 Hours, 15 marks)*

Module III

Sequence, Series and Progression : Concepts and differences - Arithmetic progression- n th term and sum of n terms of an AP - Insertion of Arithmetic means in AP - Geometric progression- ‗n‘th term and sum of n terms of an GP - Insertion of Geometric Mean in GP – Harmonic progression. *(20 Hours, 15 marks*)

Module IV

Interest and Time value: Concept of interest-Types of interest: Simple interest and compound interest – nominal, real and effective rate of interest - Future value and Present Value; Annuity and Perpetuity - Computing future and present values of annuity (regular and immediate) - multi and growing period perpetuity - Compound annual growth rate - computation of Equated Monthly Installments (EMI) *(15 Hours, 15 marks)*

Module V

Descriptive Statistics: Measures of Central Tendency – Mean: Arithmetic mean, Geometric mean and Harmonic Mean- Median, Mode and other position values - Measures of Dispersion: mean deviation, quartile deviation, standard deviation and coefficient of variation
- Measures of Skewness and Kurtosis. *(20 Hours, 25 marks)*

<u>Reference Books</u>

1. Business Mathematics and Statistics- N G Das & J K Das (Tata McGraw Hill)
2. Basic Mathematics and its Application in Economics – S. Baruah (Macmillan)
3. Mathematics for Economics and Business – R. S. Bhardwaj (Excel Books)
4. Business Statistics – G. C. Beri (Tata McGraw Hill)
5. Fundamentals of Statistics – S.C.Gupta
(Himalaya Publishing House) 6.SP Gupta
,Statistical Methods, Sultan Chand
7. Dinesh Khattar-The Pearson guide to quantitative aptitude for competitive examinations.
8. Dr. Agarwal.R.S – Quantitative Aptitude for Competitive
Examinations, S.Chand and Company Limited.
9. Abhijit Guha, Quantitative Aptitude for Competitive Examinations, Tata Mcgraw Hill,

(Theory and problems may be in the ratio of 20% and 80% respectively. An over view of the topics is expected and only simple problems shall be given)

MODULE 1

TYPES OF MATRICES

Row Matrix:
A matrix is said to be a row matrix if it has only one row.

Column Matrix:
A matrix is said to be a column matrix if it has only one column.

Rectangular Matrix:
A matrix is said to be rectangular if the number of rows is not equal to the number of columns.

Square Matrix:
A matrix is said to be square if the number of rows is equal to the number of columns.

Diagonal Matrix:
A square matrix is said to be diagonal if at least one element of principal diagonal is non-zero and all the other elements are zero.

Scalar Matrix:
A diagonal matrix is said to be scalar if all of its diagonal elements are the same.

Identity or Unit Matrix:
A diagonal matrix is said to be identity if all of its diagonal elements are equal to one, denoted by I.

Triangular Matrix:
A square matrix is said to be triangular if all of its elements above the principal diagonal are zero **(lower triangular matrix)** or all of its elements below the principal diagonal are zero **(upper triangular matrix)**.

Null or Zero Matrix:
A matrix is said to be a null or zero matrix if all of its elements are equal to zero. It is denoted by O.

Transpose of a Matrix:
Suppose A is a given matrix, then the matrix obtained by interchanging its rows into columns is called the transpose of A

MATRIX OPERATION

1. **ADDITION**

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & 3 \\ 7 & & 9 \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} 5 & 6 & 7 \\ 3 & 4 & 5 \end{bmatrix}$$

$$\mathbf{A} + \mathbf{B} = \begin{bmatrix} 1+5 & 2+6 & 3+ \\ 7+3 & 8+4 & 9+ \end{bmatrix}$$

$$\mathbf{A} + \mathbf{B} = \begin{bmatrix} 6 & 8 & 10 \\ 10 & 12 & 14 \end{bmatrix}$$

And,

$$\mathbf{A} - \mathbf{B} = \begin{bmatrix} 1-5 & 2-6 & 3-7 \\ 7-3 & 8-4 & 9-5 \end{bmatrix}$$

$$\mathbf{A} - \mathbf{B} = \begin{bmatrix} -4 & -4 & -4 \\ 4 & 4 & 4 \end{bmatrix}$$

2. **MULTIPLICATION BY SCALAR**

For the following matrix A, find $2A$

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & 3 \\ 7 & 8 & 9 \end{bmatrix}$$

$$\mathbf{2A} = \begin{bmatrix} 2 & 4 & 6 \\ 14 & 16 & 18 \end{bmatrix}$$

DETERMINANT

Determinant is a scalar value that can be computed from the elements of a square matrix and encodes certain properties of the linear transformation described by the matrix. The determinant of a matrix A is Denoted $\det(A)$, $\det A$, or $|A|$.

$$C = \begin{bmatrix} 6 & 1 & 1 \\ 4 & -2 & 5 \\ 2 & 8 & 7 \end{bmatrix}$$

$$|C| = 6\times(-2\times7 - 5\times8) - 1\times(4\times7 - 5\times2) + 1\times(4\times8 - (-2\times2))$$
$$= 6\times(-54) - 1\times(18) + 1\times(36)$$
$$= \mathbf{-306}$$

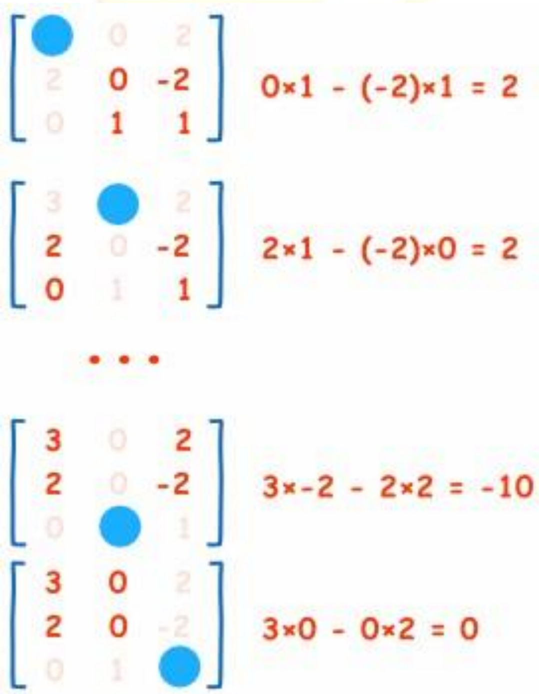
$$B = \begin{bmatrix} 4 & 6 \\ 3 & 8 \end{bmatrix}$$

$$|B| = 4\times8 - 6\times3$$
$$= 32 - 18$$
$$= 14$$

COOFACTOR

A ***cofactor*** is the number you get when you remove the column and row of a designated element in a ***matrix*** ..

$$A = \begin{bmatrix} 3 & 0 & 2 \\ 2 & 0 & -2 \\ 0 & 1 & 1 \end{bmatrix}$$



Definition of Adjoint of a Matrix

The adjoint of a square matrix $A = [a_{ij}]_{n \times n}$ is defined as the transpose of the matrix $[A_{ij}]_{n \times n}$, where Aij is the cofactor of the element a_{ij}. Adjoing of the matrix A is denoted by **adj A**.

$$\text{Thus if } A = \begin{bmatrix} a_{11} & a_{12} & \cdots\cdots & a_{1n} \\ a_{21} & a_{22} & \cdots\cdots & a_{2n} \\ \cdots\cdots & \cdots\cdots & \cdots\cdots & \cdots \\ a_{n1} & a_{n2} & \cdots\cdots & a_{nn} \end{bmatrix}$$

$$\text{Adj}.A = \text{transpose of} \begin{bmatrix} A_{11} & A_{12} & \cdots\cdots & A_{1n} \\ A_{21} & A_{22} & \cdots\cdots & A_{2n} \\ A_{n1} & A_{n2} & \cdots\cdots & A_{nn} \end{bmatrix}$$

$$= \begin{bmatrix} A_{11} & A_{21} & \cdots & A_{n1} \\ A_{12} & A_{22} & \cdots & A_{n2} \\ . & & & \\ . & & & \\ . & & & \\ A_{1n} & A_{12} & \cdots & A_{nn} \end{bmatrix}$$

Find the adjoint of the matrix:

$$\begin{bmatrix} 1 & -1 & 2 \\ 4 & 0 & 6 \\ 0 & 1 & -1 \end{bmatrix}$$

Solution: We will first evaluate the cofactor of every element,

$$\text{cof}(a_{11}) = +\begin{vmatrix} 0 & 6 \\ 1 & -1 \end{vmatrix} = -6 \qquad \text{cof}(a_{12}) = -\begin{vmatrix} 4 & 6 \\ 0 & -1 \end{vmatrix} = 4$$

$$\text{cof}(a_{21}) = -\begin{vmatrix} -1 & 2 \\ 1 & -1 \end{vmatrix} = 1 \qquad \text{cof}(a_{22}) = +\begin{vmatrix} 1 & 2 \\ 0 & -1 \end{vmatrix} = -1$$

$$\text{cof}(a_{31}) = +\begin{vmatrix} -1 & 2 \\ 0 & 6 \end{vmatrix} = -6 \qquad \text{cof}(a_{32}) = -\begin{vmatrix} 1 & 2 \\ 4 & 6 \end{vmatrix} = 2$$

$$\text{cof}(a_{13}) = +\begin{vmatrix} 4 & 0 \\ 0 & 1 \end{vmatrix} = 4$$

$$\text{cof}(a_{23}) = -\begin{vmatrix} 1 & -1 \\ 0 & 1 \end{vmatrix} = -1$$

$$\text{cof}(a_{33}) = +\begin{vmatrix} 1 & -1 \\ 4 & 0 \end{vmatrix} = 4$$

Therefore,

$$\text{Adj}\,A = [\text{cof}(a_{ij})]^T = \begin{bmatrix} -6 & 4 & 4 \\ 1 & -1 & -1 \\ -6 & 2 & 4 \end{bmatrix}^T = \begin{bmatrix} -6 & 1 & -6 \\ 4 & -1 & 2 \\ 4 & -1 & 4 \end{bmatrix}$$

The **inverse** of A is A^{-1} only when $A \times A^{-1} = A^{-1} \times A = I$

Rank of matrix

The **rank** of a **matrix** is defined as (a) the maximum number of linearly independent column vectors in the **matrix** or (b) the maximum number of linearly independent row vectors in the **matrix**

Find the rank of the matrix $\begin{pmatrix} 1 & 5 \\ 3 & 9 \end{pmatrix}$

Solution:

Let A= $\begin{pmatrix} 1 & 5 \\ 3 & 9 \end{pmatrix}$

Order of A is 2×2 ∴

$\rho(A)$

≤ 2 Consider the second order

minor

$\begin{vmatrix} 1 & 5 \\ 3 & 9 \end{vmatrix} = -6 \neq 0$

There is a minor of order 2, which is not zero. ∴$\rho(A) = 2$

Cramer's Rule

Cramer's Rule is an explicit formula for the solution of a system of linear equations with as many equations as unknowns,

Crammer's Rule 2x2

$$x - y = 4 \qquad x = ? \qquad x = \frac{D_x}{D} \qquad y = \frac{D_y}{D}$$
$$2x + y = 2 \qquad y = ?$$

$$D = \begin{vmatrix} 1 & -1 \\ 2 & 1 \end{vmatrix} = 3 \qquad x = \frac{D_x}{D} = \frac{6}{3} = 2$$

$$D_x = \begin{vmatrix} 4 & -1 \\ 2 & 1 \end{vmatrix} = 6 \qquad y = \frac{D_y}{D} = \frac{-6}{3} = -2$$

$$D_y = \begin{vmatrix} 1 & 4 \\ 2 & 2 \end{vmatrix} = -6$$

MODULE 2

Types of Equations :

- Quadratic Equation
- Linear Equation
- Radical Equation
- Exponential Equation
- Rational Equation

Linear Equations

- Each term involved in the linear equation is either a constant or single variable or a product of a constant. The general form of linear equations with two variables is given by
- Y = mx + c ,m≠0

Quadratic Equations

- The quadratic equation is a second-order equation in which any one of the variable contains an exponent of 2. The general form of the quadratic equation is
- $ax^2+bx+c = 0$, a≠0

Radical Equations

- In a radical equation, a variable is lying inside a square root symbol or you can say that the maximum exponent on a variable is ½
- Example : a−−√+10=26

Exponential Equations

- In this math equations, it contains the variables in place of exponents. By using the property, an exponential equation can be solved. ax=ay⇒x=y

Rational Equations

- A rational math equations involves the rational expressions
- Example : y2=y+24

Simple linear equations in one unknown

x - 5 = 2
x - 5 + 5 = 2 + 5
x = 7

Simultaneous equations of first degree in two unknows

Solve for ‘x’ and ‘y’

$$2x + y = 9 \text{ .... (i)}$$

$$x + 2y = 21 \text{ .... (ii)}$$

Solution:

Using method of substitution:

From equation (i) we get, y =

$9 - 2x$

Substituting value of ‘y’ from equation (i) in equation (ii):

$x + 2(9 – 2x) = 21$

$\Rightarrow x + 18 – 4x = 21$

$\Rightarrow -3x = 21 – 18$

$\Rightarrow -3x = 3$

$\Rightarrow -x = 1$

$\Rightarrow x = -1$

Substituting x = -1 in equation 2:

$y = 9 – 2(-1)$

$= 9 + 2$

$= 11.$

Hence x = -1 and y = 11.

This method is known as method of substitution.

Quadratic equations

Solve for x: $x^2-3x-10 = 0$

Solution:
Let us express -3x as a sum of -5x and +2x.

$\rightarrow x^2-5x+2x-10 = 0$

$\rightarrow x(x-5)+2(x-5) = 0$

$\rightarrow (x-5)(x+2) = 0$

$\rightarrow x-5 = 0$ or $x+2 = 0$

$\rightarrow x = 5$ or $x = -2$

Simultaneous equations of two unknown when one of them is quadratic and other is linear

Solve the following simultaneous equations:

$$y = x+2$$
$$y = x^2$$

Substitute $y = x^2$ in (1):

$$x^2 = x+2$$
$$x^2 - x - 2 = 0$$
$$(x-2)(x+1) = 0$$
$$\therefore x-2 = 0 \quad \text{or} \quad x+1 = 0$$
$$x = 2 \quad \text{or} \quad x = -1$$

When $x = 2,\ y = 2^2$
$= 4$

When $x = -1,\ y = (-1)^2$
$= 1$

So, the solution set is $\{(-1, 1), (2, 4)\}$.

$$y = x+2 \quad \ldots(1)$$
$$y = x^2 \quad \ldots(2)$$

Simultaneous equations containing three unknowns

$$\begin{cases} 2x - y + z = 10 \\ 4x + 2y - 3z = 10 \\ x - 3y + 2z = 8 \end{cases}$$

Step 1: Select equation 1 and 2, then multiply the first equation by 2

$$(2)\begin{cases} 2x - y + z = 10 \\ 4x + 2y - 3z = 10 \end{cases}$$

$$\begin{cases} 4x - 2y + 2z = 20 \\ 4x + 2y - 3z = 10 \end{cases}$$

$$\mathbf{8x - z = 30}$$

Step 2: Select eq.1 and 3

$$(-3)\begin{cases} 2x - y + z = 10 \\ x - 3y + 2z = 8 \end{cases}$$

$$\begin{cases} -6x + 3y - 3z = -30 \\ x - 3y + 2z = 8 \end{cases}$$

$$\mathbf{-5x - z = -22}$$

Step 3: combine

$$\begin{cases} 8x - z = 30 \\ -5x - z = -22 \end{cases}$$

should get $x = 4$, $z = 2$

Step 4: Substitute to

$$2x - y + z = 10$$

$$8 - y + 2 = 10$$

$$10 - y = 10$$

$$y = 0$$

Solution:

(4, 0, 2)

MODULE 3

Sequence

a sequence is a list of objects (or events) which have been ordered in a sequential fashion

Series

A series is a sum of a sequence of terms.

Arithmetic Progression

An arithmetic progression is a sequence of numbers such that the difference of any two successive members is a constant

The nth term of an AP $T_n = a + (n-1)d$

Given AP is 20,16,12.............[-176]
Here a = 20, d = 16 − 20 = − 4
$t_n = -176$
nth term of an AP is $t_n = a + (n - 1)d$
$\Rightarrow -176 = 20 + (n - 1)(-4)$
$\Rightarrow -176 = 20 - 4n + 4$
$\Rightarrow -176 = 24 - 4n$
$\Rightarrow -176 - 24 = -4n$
$\Rightarrow -200 = -4n$
$\therefore n = 50$
The middle terms are 25th and 26th terms
$t_{25} = 20 + (25 - 1)(-4)$
$= 20 - 96 = -76$
$t_{26} = 20 + (26 - 1)(-4)$
$= 20 - 100 = -80$

Sum of n terms of an AP.

$S_n = n/2[2a+(n-1)d]$

$a=12$, $d=10$, $n=20$

$$S_n = \frac{1}{2}n(2a+(n-1)d)$$

$$\Rightarrow \quad S_{20} = \frac{1}{2}\times 20(2\times 12+(20-1)\times 10)$$

$$\Rightarrow \quad S_{20} = \frac{1}{2}\times 20(24+190)$$

$$\Rightarrow \quad S_{20} = \frac{1}{2}\times 20\times 214$$

$$\Rightarrow \quad S_{20} = 2140$$

Arithmetic Mean

The average of a set of numerical values, as calculated by adding them together and dividing by the number of terms in the set.

Geometric Progression

a geometric progression, also known as a geometric sequence, is a sequence of numbers where each term after the first is found by multiplying the previous one by a fixed, non- zero number called the common ratio. For example, the sequence 2, 6, 18, 54, ... is a geometric progression with common ratio 3

nth term of an GP

$T_n = ar^{n-1}$

Example:

1) $a_2 = 8$ **and** $a_5 = 64$ **Find** a_9

$$a_n = a_1 r^{n-1}$$

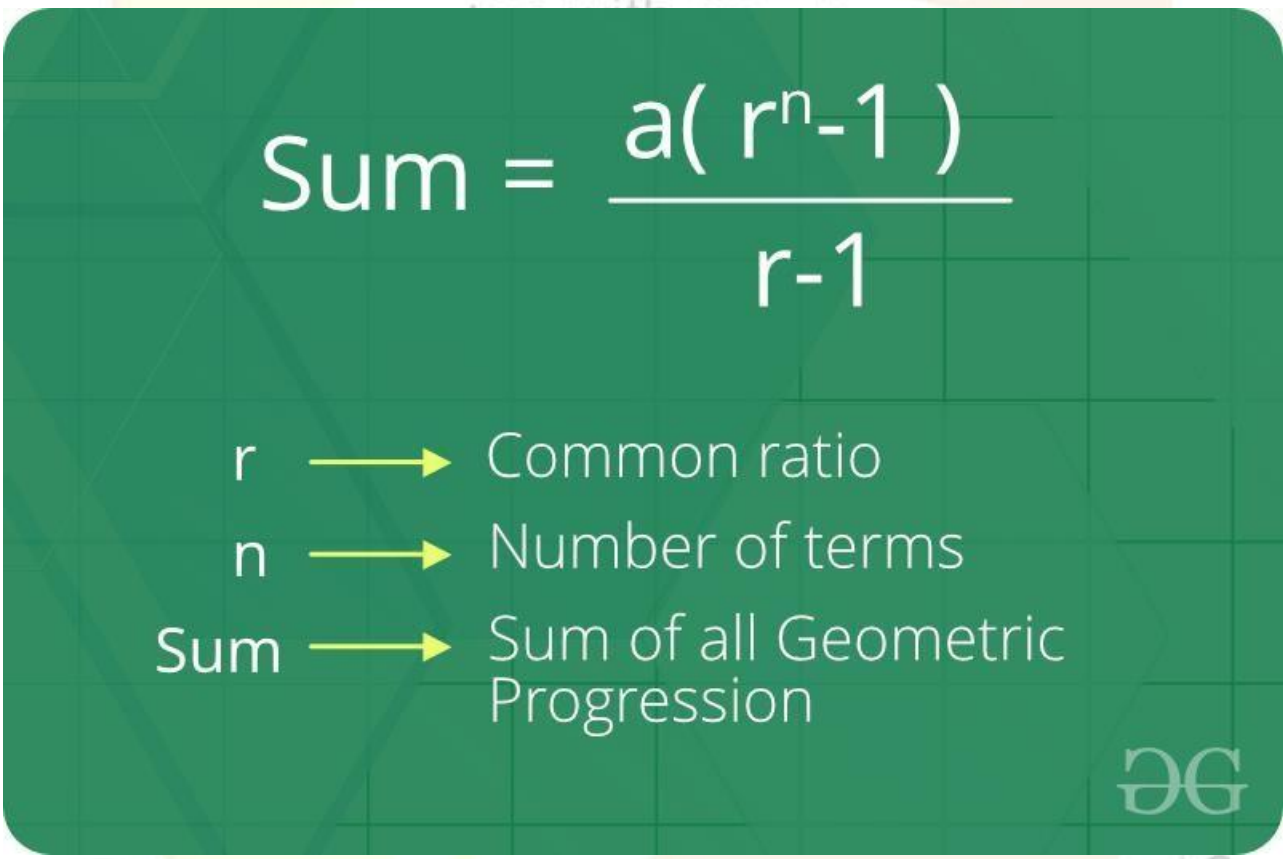
$$a_5 = a_2 r^{5-2}$$

$$64 = 8r^{5-2}$$

$$\frac{64}{8} = \frac{8r^3}{8}$$

$$8 = r^3$$

<u>Sum of n terms of a GP</u>



Geometric Mean

The geometric mean is a mean or average, which indicates the central tendency or typical value of a set of numbers by using the product of their values

Harmonic Progression

A harmonic progression is a progression formed by taking the reciprocals of an arithmetic progression

nth term of an H.P

(H.P) = 1/ [a+(n-1)d]

Example

Determine the 4th and 8th term of the harmonic progression 6, 4, 3…

Solution:

Given:
H.P = 6, 4, 3

Now, let us take the arithmetic progression from the given H.P A.P =

⅙, ¼, ⅓, ….

Here, $T_2 - T_1 = T_3 - T_2$ = 1/12 = d

So, in order to find the 4th term of an A. P, use the formula, The nth

term of an A.P = a+(n-1)d

Here, a = ⅙, d= 1/12

Now, we have to find the 4th term, So,

take n=4

Now put the values in the formula, we have 4th

term of an A.P = (⅙) +(4-1)(1/12)

= (⅙)+(3/12)

= (⅙)+ (¼)

= 5/12

Similarly, for 8th term of an A.P,

8th term of an A.P = (⅙) +(8-1)(1/12)

= (⅙)+(7/12)

= 9/12

Since H.P is the reciprocal of an A.P, we can write the values as: 4th term

of an H.P = 1/4th term of an A.P = 12/5

8th term of an H.P = 1/8th term of an A.P = 12/9 = 4/3

Harmonic Mean

the harmonic mean is one of several kinds of average, and in particular, one of the Pythagorean means.

MODULE 4

Measures Of Central Tendency.

- ✓ It also Know as Averages.
- ✓ It is a single significant Figure.
- ✓ Which sum up the characteristics of a group of figures.
- ✓ It is conveys General Idea of Whole group.
- ✓ It is Generally located at the Centre or middle of the distribution

Type of averages

1) Arithmetic Mean
2) Median
3) Mode
4) Harmonic mean
5) Geometric mean

1. Arithmetic mean

The average of a set of numerical values, as calculated by adding them together and dividing by the number of terms in the set.

Find the arithmetic mean of the first 7 natural numbers.

Solution:

The first 7 natural numbers are 1, 2, 3, 4, 5, 6 and 7.

Let x denote their arithmetic mean.

Then mean = Sum of the first 7 natural numbers/number of natural numbers

x = (1 + 2 + 3 + 4 + 5 + 6 + 7)/7

= 28/7

= 4

Hence, their mean is 4.

2. Median

The **median** is the middle number in a sorted, ascending or descending, Find the median of the following set of points in a game:

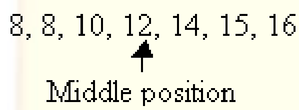
15, 14, 10, 8, 12, 8, 16

Solution:

First arrange the point values in an ascending order (or descending order).

8, 8, 10, 12, 14, 15, 16

The number of point values is 7, an odd number. Hence, the median is the value in the middle position.



Median = 12

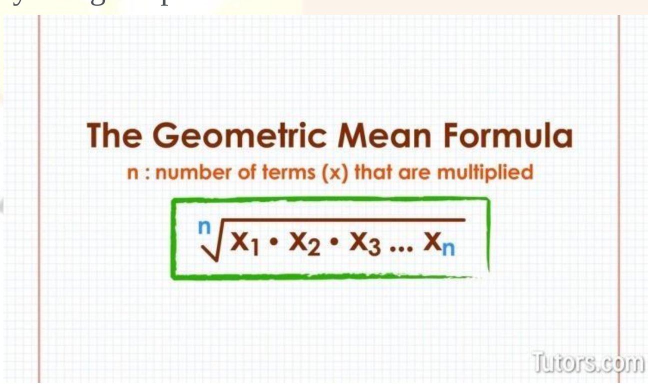
3. **Mode**

The **mode** is the number that appears most frequently in a data set

Example: in {6, 3, 9, 6, 6, 5, 9, 3} the Mode is 6 (it occurs most often).

4. **Geometric mean**

The geometric mean is a mean or average, which indicates the central tendency or typical value of a set of numbers by using the product of their values.



Example: What is the Geometric Mean of **1, 3, 9, 27 and 81**?

- First we multiply them: $1 \times 3 \times 9 \times 27 \times 81 = 59049$
- Then (as there are 5 numbers) take the 5th root: $\sqrt[5]{59049} = \mathbf{9}$

5. Harmonic mean

The harmonic mean is one of several kinds of average, and in particular, one of the Pythagorean means. Typically, it is appropriate for situations when the average of rates is desired

Example: What is the harmonic mean of 1, 2 and 4?

The reciprocals of 1, 2 and 4 are:

$$\frac{1}{1} = 1, \quad \frac{1}{2} = 0.5, \quad \frac{1}{4} = 0.25$$

Now add them up:

$$1 + 0.5 + 0.25 = 1.75$$

Divide by how many:

$$\text{Average} = \frac{1.75}{3}$$

The reciprocal of that average is our answer:

Harmonic Mean = $\frac{3}{1.75}$ = **1.714** (to 3 places)

MEASURE OF DISPERSION

1. Range
2. Quartile deviation
3. Mean deviation
4. Standard deviation

1. Range

The **range** is the difference between the largest and the smallest observation in the data.

Example: 1, 3, 5, 6, 7 => Range = 7 -1= 6

2. Quartile deviation

The **Quartile Deviation** is a simple way to estimate the spread of a distribution about a measure of its central tendency

Quartile Deviation = $(Q_3 - Q_1) / 2$
Coefficient of Quartile Deviation = $(Q_3 - Q_1) / (Q_3 + Q_1)$

Example:-

Consider a data set of following numbers: 22, 12, 14, 7, 18, 16, 11, 15, 12. You are required to calculate the Quartile Devi Solution:

First, we need to arrange data in ascending order to find Q3 and Q1 and avoid any duplicates.

7, 11, 12, 13, 14, 15, 16, 18, 22

Calculation of Q1 can be done as follows,

Q1 = ¼ (9 + 1)

=¼ (10)

Q1=**2.5 Term**

Calculation of Q3 can be done as follows,

Q3=¾ (9 + 1)

=¾ (10)

Q3= **7.5 Term**

3. Mean deviation

The mean of the absolute values of the numerical differences between the numbers of a set (such as statistical data) and their mean or median

Calculation of MD

Individual Series	Discrete Series	Continuous Series
∑ /d/ n	∑ f/d/ N	∑ f/d/ N

Example: the Mean Deviation of 3, 6, 6, 7, 8, 11, 15, 16

Step 1: Find the **mean**:

Mean = 3 + *6* + *6* + *7* + *8* + *11* + *15* + *16***8** = 728 = 9

Step 2: Find the **distance** of each value from that mean:

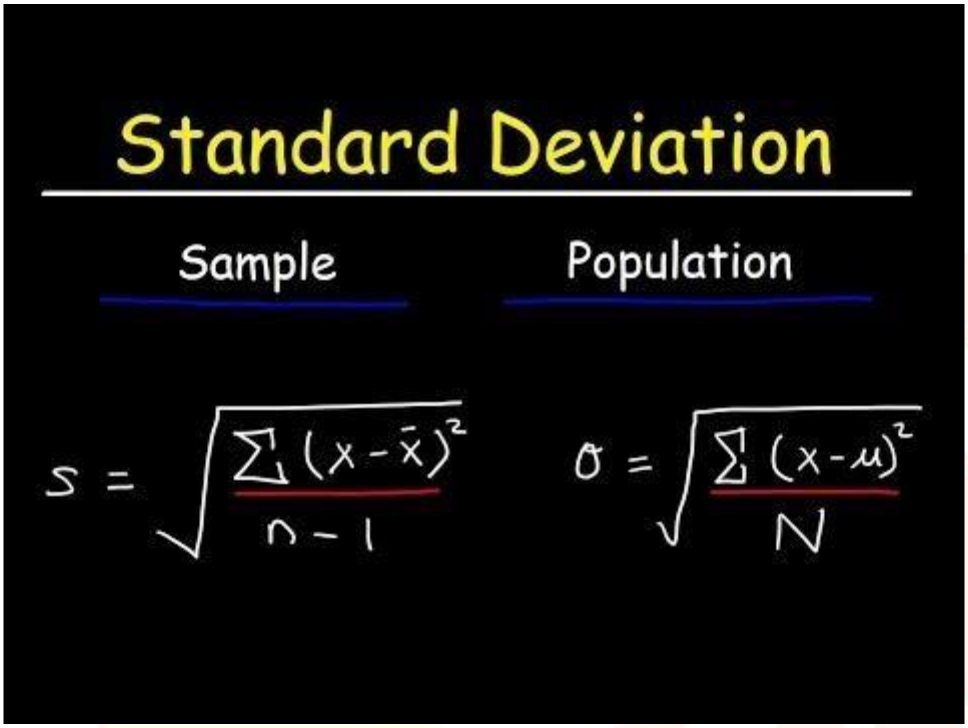
Value	Distance from 9
3	6
6	3
6	3
7	2
8	1
11	2
15	6
16	7

Step 3. Find the **mean of those distances**:

Mean Deviation = 6 + *3* + *3* + *2* + *1* + *2* + *6* + *7***8** = 308 = **3.75**

4. Standard deviation

It is the square root of the mean of the square of the deviations of all values of a series from their arithmetic mean



9, 2, 5, 4, 12, 7, 8, 11, 9, 3, 7, 4, 12, 5, 4, 10, 9, 6, 9, 4

Calculate the sample standard deviation of the length of the crystals.

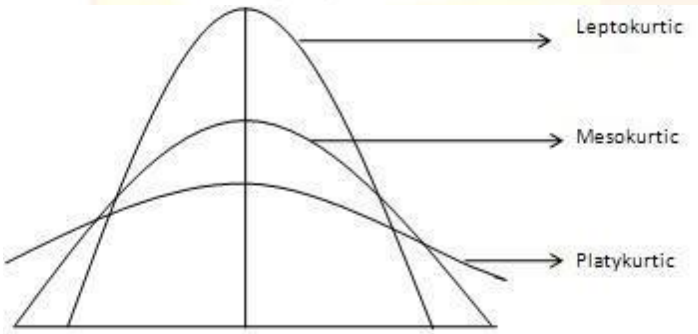
1. Calculate the mean of the data. Add up all the numbers and divide by the total number of data points.(9+2+5+4+12+7+8+11+9+3+7+4+12+5+4+10+9+6+9+4) / 20 = 140/20 = 7

2. Subtract the mean from each data point (or the other way around, if you prefer... you will be squaring this number, so it does not matter if it is positive or negative).$(9 - 7)^2 = (2)^2 = 4$
   $(2 - 7)^2 = (-5)^2 = 25$
   $(5 - 7)^2 = (-2)^2 = 4$
   $(4 - 7)^2 = (-3)^2 = 9$
   $(12 - 7)^2 = (5)^2 = 25$
   $(7 - 7)^2 = (0)^2 = 0$
   $(8 - 7)^2 = (1)^2 = 1$
   $(11 - 7)^2 = (4)2^2 = 16$
   $(9 - 7)^2 = (2)^2 = 4$
   $(3 - 7)^2 = (-4)2^2 = 16$
   $(7 - 7)^2 = (0)^2 = 0$
   $(4 - 7)^2 = (-3)^2 = 9$
   $(12 - 7)^2 = (5)^2 = 25$
   $(5 - 7)^2 = (-2)^2 = 4$
   $(4 - 7)^2 = (-3)^2 = 9$
   $(10 - 7)^2 = (3)^2 = 9$
   $(9 - 7)^2 = (2)^2 = 4$
   $(6 - 7)^2 = (-1)^2 = 1$
   $(9 - 7)^2 = (2)^2 = 4$
   $(4 - 7)^2 = (-3)2^2 = 9$

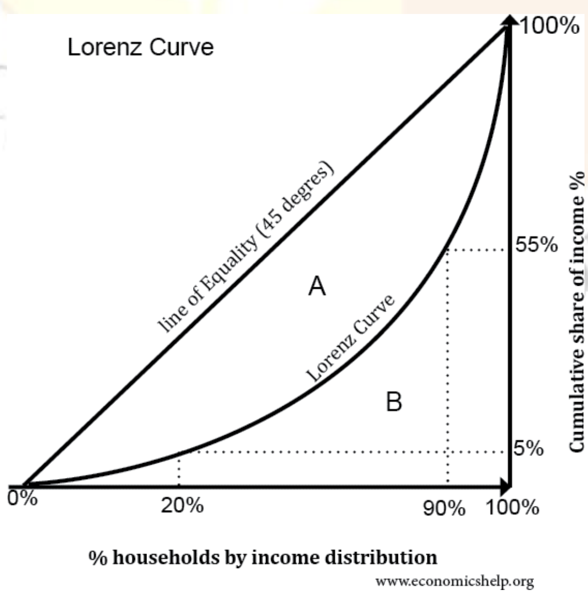
3. Calculate the mean of the squared differences.(4+25+4+9+25+0+1+16+4+16+0+9+25+4+9+9+4+1+4+9) / 19 = 178/19 = 9.368 This value is the **sample variance**. The sample variance is 9.368

4. The population standard deviation is the square root of the variance. Use a calculator to obtain this number.$(9.368)^{1/2}$ = 3.061The population standard deviation is 3.061

Measure of kurtosis



Lorenz curve



MODULE 5

SIMPLE INTEREST

When the interest is calculated on principal at a uniform rate every period, it is called simple interest

Simple interest = **pnr/100 p-**

Principal amount

n- Number of years

r- Rate of interest per annum

Example: Calculate the Simple Interest if the principal amount is Rs. 2000, the time period is 1 year and the rate is 10%. Also, calculate the total amount at the end of 1 year.

Solution:

According to the formula of simple interest we have,

S.I. = [(Principal (P) × Time (T) × Rate (r)) / 100] So, from the above values,

S.I. = [(2000 × 1 × 10)] / 100

= 20000/100

=200

So, the simple interest at the end of 1 year will be Rs. 200. For the amount after 1 year,

A = P + S.I.

So, A = 2000+200 = 2200

Hence, the total amount at the end of the given tenure (i.e. 1 year) will be Rs. 2200.

COMPOUND INTEREST

Compound interest is the addition of interest to the principal sum of a loan or deposit A= P(1+r/100)n

Example: A sum of **Rs.10000** is borrowed by Akshit for **2** years at an interest of **10%** compounded annually. Calculate the compound interest and amount he has to pay at the end of **2** years.

Solution:

Given,

Principal/ Sum = Rs.

10000, Rate = 10%, and

Time = 2 years

Amount (A2) = P (1+R100)2
A2= =10000(1+10100)2
10000(1110)(1110)=Rs.1210
0
Compound Interest (for 2nd year) = A2–P = 12100 – 10000 = Rs. 2100

CONTINUOUS COMPOUNDING

A=P (1+i) n
Where i= r/100
Example: An amount of Rs. 2340.00 is deposited in a bank paying an annual interest rate of 3.1%, compounded continuously. Find the balance after 3 years.

Solution:

Use the continuous compound interest formula,

Given P = 2340

r = (3.1 / 100) = 0.031

t = 3

Use the continuous compound interest formula, A

$= Pe^{rt}$

Given,

P = 2340

r = 3.1 = (3.1 / 100) = 0.031

t = 3

Here: e stands for the Napier's number, which are approximately 2.7183.

However, one does not have to plug this value in the formula, as the calculator has a built-in key for e. Therefore,

$A = 2340\ e^{0.031\ (3)} \approx 2568.06$

So, the balance after 3 years is approximately Rs. 2,568.06.

ANNUITY

An annuity is a series of payments made at equal intervals

$$\textbf{PV of an Ordinary Annuity} = \mathbf{R} * \frac{\mathbf{1 - (1 + i)^{-n}}}{\mathbf{i}}$$

$$\textbf{PV of an Annuity Due} = \mathbf{R} * \frac{\mathbf{1 - (1 + i)^{-n}}}{\mathbf{i}} * \mathbf{(1 + i)}$$

EQUATED MONTHLY INSTALMENTS (EMI)

EMI = P+(P x n x i)/n x
12 p= loan amount
n= number of years
i= r/100 (rate of interest)