**4th sem BCOM**

**CALICUT UNIVERSITY**

# QUANTITATIVE TECHNIQUES

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MODULE 1 **QUANTITATIVE TECHNIQUES**

Meaning and Definition:

- Quantitative techniques may be defined as those techniques which provide the decision makes a systematic and powerful means of analysis, based on quantitative data.
- It is a scientific method employed for problem solving and decision making by the management.

Classification of Quantitative Techniques:

1. Mathematical Quantitative Techniques
2. Statistical Quantitative Techniques
3. Programming Quantitative Techniques

## I. Mathematical quantitative techniques

1. Permutations and Combinations:
   Permutation means arrangement of objects in a definite order.

$$\mathrm{nPr} = \frac{\mathrm{n!}}{(n-r)!}$$

Combination means selection or grouping objects without considering their order.

$$\mathrm{nCr} = \frac{\mathrm{n!}}{(n-r)!\mathrm{r!}}$$

2. Set Theory:
   Set theory is a modern mathematical device which solves various types of critical problems.
3. Matrix Algebra:
   It is a mathematical device of finding out the results of different types of algebraic operations on the basis of the relevant matrices.
4. Determinants:
   This device is used for finding out values of different variables connected with a number of simultaneous equations.
5. Differentiation:
   It is a mathematical process of finding out changes in the dependent variable with reference to a small change in the independent variable.
6. Integration:
   Integration is the reverse process of differentiation.
7. Differential Equation:
   It is a mathematical equation which involves the differential coefficients of the dependent variables.

## II. Statistical Quantitative Techniques:

Statistical techniques are those techniques which are used in conducting the statistical enquiry concerning to certain Phenomenon.

1. Collection of data:
   There are different methods for collecting primary and secondary data.
2. Measures of Central tendency, dispersion, skewness and Kurtosis:
   - Measures of Central tendency is a method used for finding he average of a series
   - Measures of dispersion used for finding out the variability in a series.
   - Measures of Skewness measures asymmetry of a distribution
   - Measures of Kurtosis measures the flatness of peakedness in a distribution.
3. Correlation and Regression Analysis:
   - Correlation is used to study the degree of relationship among two or more variables.
   - Regression technique is used to estimate the value of one variable for a given value of another.
4. Index Numbers:
   Index numbers measure the fluctuations in various Phenomena like price, production etc. over a period of time, They are described as economic barometres.
5. Time series Analysis:
   Analysis of time series helps us to know the effect of factors which are responsible for changes:
6. Interpolation and Extrapolation:
   - Interpolation is the statistical technique of estimating under certain assumptions, the missing figures which may fall within the range of given figures.
   - Extrapolation provides estimated figures outside the range of given data.
7. Statistical Quality Control:
   Statistical quality control is used for ensuring the quality of items manufactured.
8. Ratio Analysis:
   Ratio analysis is used for analyzing financial statements of any business or industrial concerns which help to take appropriate decisions.
9. Probability Theory:
   Theory of probability provides numerical values of the likely hood of the occurrence of events.
10. Testing of Hypothesis:
    Testing of hypothesis is a tool to judge the reliability of inferences drawn on the basis of sample studies.

## III. Programming Techniques

Programming techniques are also called operations research techniques.

1. Linear Programming:
   Linear programming technique is used in finding a solution for optimizing a given objective under certain constraints.
2. Queuing Theory:
   Queuing theory deals with mathematical study of queues. It aims at minimizing cost of both servicing and waiting.
3. Game Theory:
   Game theory is used to determine the optimum strategy in a competitive situation.
4. Decision Theory:
   This is concerned with making sound decisions under conditions of certainty, risk and uncertainty.
5. Inventory Theory:
   Inventory theory helps for optimizing the inventory levels. It focuses on minimizing cost associated with holding of inventories.
6. Network programming:
   It is a technique of planning, scheduling, controlling, monitoring and coordinating large and complex projects comprising of a number of activities and events. It includes CPM, PERT etc.
7. Simulation:
   It is a technique of testing a model which resembles real life situations .
8. Replacement Theory:
   - It is concerned with the problems of replacement of machines, etc due to their deteriorating efficiency or breakdown.
   - It helps to determine the most economic replacement policy.
9. Non Linear Programming:
   It is a programming technique which involves finding an optimum solution to a problem in which some or all variables are non-linear.
10. Quadratic Programming:
    Quadratic programming technique is designed to solve certain problems, the objective function of which takes the form of a quadratic equation.
11. Branch and Bound Technique :
    This is designed to solve the combinational problems of decision making where there is large number of feasible solutions.
12. Search theory:
    Search theory concerns with research problems.
13. Dynamic programming:
    Dynamic programming is applied to solve certain problems by dividing the total problems into a number of subsidiary problems.
14. Integer programming:
    It is a technique to solve linear programming problems in which the solution values are expressed in whole numbers.

15. Parametric programming:
    It is a technique designed to solve a problem consisting of more than one objective function with varying degrees of priorities.
16. Markov process:
    It is a technique in which various states of a problem are defined and the steady state probabilities of going to each of the states from a given state are calculated.

Functions of Quantitative Techniques:

1. To facilitate the decision-making process
2. To provide tools for scientific research
3. To help in choosing an optimal strategy
4. To enable in proper deployment of resources
5. To help in minimizing costs
6. To help in minimizing the total processing time required for performing a set of jobs

USES OF QUANTITATE TECHNIQUES BUSINESS AND INDUSTRY

1. Quantitative techniques of linear programming is used for optimal allocation of scarce resources in the problem of determining product mix .

2. Inventory control techniques are useful in dividing when and how much items are to be purchase so as to maintain a balance between the cost of holding and cost of ordering the inventory

3. Quantitative techniques of CPM, and PERT helps in determining the earliest and the latest times for the events and activities of a project. This helps the management in proper deployment of resources.

4. Decision tree analysis and simulation technique help the management in taking the best possible course of action under the conditions of risks and uncertainty.

5. Queuing theory is used to minimize the cost of waiting and servicing of the customers in queues.
6. Replacement theory helps the management in determining the most economic replacement policy regarding replacement of equipment.

Limitations of Quantitative Techniques:

1. Quantitative techniques involves mathematical models, equations and other mathematical expressions.

2. Quantitative techniques are based on number of assumptions. Therefore, due care must be ensured while using quantitative techniques, otherwise it will lead to wrong conclusions.

3. Quantitative techniques are very expensive.

4. Quantitative techniques do not take into consideration intangible facts like skill, attitude etc.

5. Quantitative techniques are only tools for analysis and decision-making. They are not decisions itself.

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# CORRELEATION ANALYSIS

## Introduction:

- In practice, we may come across with lot of situations which need statistical analysis of either one or more variables. The data concerned with one variable only is called univariate data. For Example: Price, income, demand, production, weight, height marks etc are concerned with one variable only. The analysis of such data is called univariate analysis.
- The data concerned with two variables are called bivariate data. For example: rainfall and agriculture; income and consumption; price and demand; height and weight etc. The analysis of these two sets of data is called bivariate analysis.
- The date concerned with three or more variables are called multivariate date. For example: agricultural production is influenced by rainfall, quality of soil, fertilizer etc.

## Definition:

Two or more variables are said to be correlated if the change in one variable results in a corresponding change in the other variable.

According to Simpson and Kafka, ―Correlation analysis deals with the association between two or more variables‖.

Lun chou defines, ―Correlation analysis attempts to determine the degree of relationship between variables‖.

Boddington states that ―Whenever some definite connection exists between two or more groups or classes of series of data, there is said to be correlation.‖

## Correlation Coefficient:

Correlation analysis is actually an attempt to find a numerical value to express the extent of relationship exists between two or more variables. The numerical measurement showing the degree of correlation between two or more variables is called correlation coefficient. Correlation coefficient ranges between -1 and +1.

## SIGNIFICANCE OF CORRELATION ANALYSIS

Correlation analysis is of immense use in practical life because of the following reasons:

1. Correlation analysis helps us to find a single figure to measure the degree of relationship exists between the variables.
2. Correlation analysis helps to understand the economic behavior
3. Correlation analysis enables the business executives to estimate cost, price and other variables.
4. Correlation analysis can be used as a basis for the study of regression. Once we know that two variables are closely related, we can estimate the value of one variable if the value of other is known.
5. Correlation analysis helps to reduce the range of uncertainty associated with decision making. The prediction based on correlation analysis is always near to reality.
6. It helps to know whether the correlation is significant or not. This is possible by comparing the correlation co-efficient with 6PE. It _r' is more than 6 PE, the correlation is significant.

## Classification of Correlation

Correlation can be classified in different ways. The following are the most important classifications

1. Positive and Negative correlation
2. Simple, partial and multiple correlation
3. Linear and Non-linear correlation

## Positive and Negative Correlation

### Positive Correlation

When the variables are varying in the same direction, it is called positive correlation. In other words, if an increase in the value of one variable is accompanied by an increase in the value of other variable or if a decrease in the value of one variable is accompanied by a decree se in the value of other variable, it is called positive correlation.

| | | | | | | |
|---|---|---|---|---|---|---|
| Eg: 1) | A: 10 | 20 | 30 | 40 | 50 |
| | B: 80 | 100 | 150 | 170 | 200 |

When the variables are moving in opposite direction, it is called negative correlation. In other words, if an increase in the value of one variable is accompanied by a decrease in the value

of other variable or if a decrease in the value of one variable is accompanied by an increase in the value of other variable, it is called negative correlation.

## Simple, Partial and Multiple correlation

### Simple Correlation

In a correlation analysis, if only two variables are studied it is called simple correlation. Eg. the study of the relationship between price & demand, of a product or price and supply of a product is a problem of simple correlation.

### Multiple correlation

In a correlation analysis, if three or more variables are studied simultaneously, it is called multiple correlations. For example, when we study the relationship between the yield of rice with both rainfall and fertilizer together, it is a problem of multiple correlation.

### Partial correlation

In a correlation analysis, we recognize more than two variable, but consider one dependent variable and one independent variable and keeping the other Independent variables as constant. For example yield of rice is influenced b the amount of rainfall and the amount of fertilizer used. But if we study the correlation between yield of rice and the amount of rainfall by keeping the amount of fertilizers used as constant, it is a problem of partial correlation.

## Linear and Non-linear correlation

### Linear Correlation

In a correlation analysis, if the ratio of change between the two sets of variables is same, then it is called linear correlation.

For example when 10% increase in one variable is accompanied by 10% increase in the other variable, it is the problem of linear correlation.

| | | | | |
|---|---|---|---|---|
| X: | 10 | 15 | 30 | 60 |
| Y: | 50 | 75 | 150 | 300 |

Here the ratio of change between X and Y is the same. When we plot the data in graph paper, all the plotted points would fall on a straight line.

### Non-linear correlation

In a correlation analysis if the amount of change in one variable does not bring the same ratio of change in the other variable, it is called nonlinear correlation.

| | | | | | |
|---|---|---|---|---|---|
| X: | 2 | 4 | 6 | 10 | 15 |
| Y: | 8 | 10 | 18 | 22 | 26 |

Here the change in the value of X does not being the same proportionate change in the value of Y

**Degrees of correlation**:

Correlation exists in various degrees

1. **Perfect positive correlation**

If an increase in the value of one variable is followed by the same proportion of increase in other related variable or if a decrease in the value of one variable is followed by the same proportion of decrease in other related variable, it is perfect positive correlation. eg: if 10% rise in price of a commodity results in 10% rise in its supply, the correlation is perfectly positive.
Similarly, if 5% full in price results in 5% fall in supply, the correlation is perfectly positive.

2. **Perfect Negative correlation**

If an increase in the value of one variable is followed by the same proportion of decrease in other related variable or if a decrease in the value of one variable is followed by the same proportion of increase in other related variably it is Perfect Negative Correlation. For example if 10% rise in price results in 10% fall in its demand the correlation is perfectly negative. Similarly if 5% fall in price results in 5% increase in demand, the correlation is perfectly negative.

3. **Limited Degree of Positive correlation**:

When an increase in the value of one variable is followed by a non-proportional increase in other related variable, or when a decrease in the value of one variable is followed by a non-proportional decrease in other related variable, it is called limited degree of positive correlation.

For example, if 10% rise in price of a commodity results in 5% rise in its supply, it is limited degree of positive correlation. Similarly if 10% fall in price of a commodity results in 5% fall in its supply, it is limited degree of positive correlation.

4. **Limited degree of Negative correlation**

When an increase in the value of one variable is followed by a non-proportional decrease in other related variable, or when a decrease in the value of one variable is followed by a non-proportional increase in other related variable, it is called limited degree of negative correlation.

For example, if 10% rise in price results in 5% fall in its demand, it is limited degree of negative correlation. Similarly, if 5% fall in price results in 10% increase in demand, it is limited degree of negative correlation.

5. **Zero Correlation (Zero Degree correlation)**

If there is no correlation between variables it is called zero correlation. In other words, if the values of one variable cannot be associated with the values of the other variable, it is zero correlation.

**Methods of measuring correlation** Correlation between 2 variables can be measured by graphic methods and algebraic methods.

I Graphic Methods

1) Scatter Diagram

2)

II Algebraic methods (Mathematical methods or statistical methods or Co-efficient of correlation methods):

1) Karl Pearson's Co-efficient of correlation

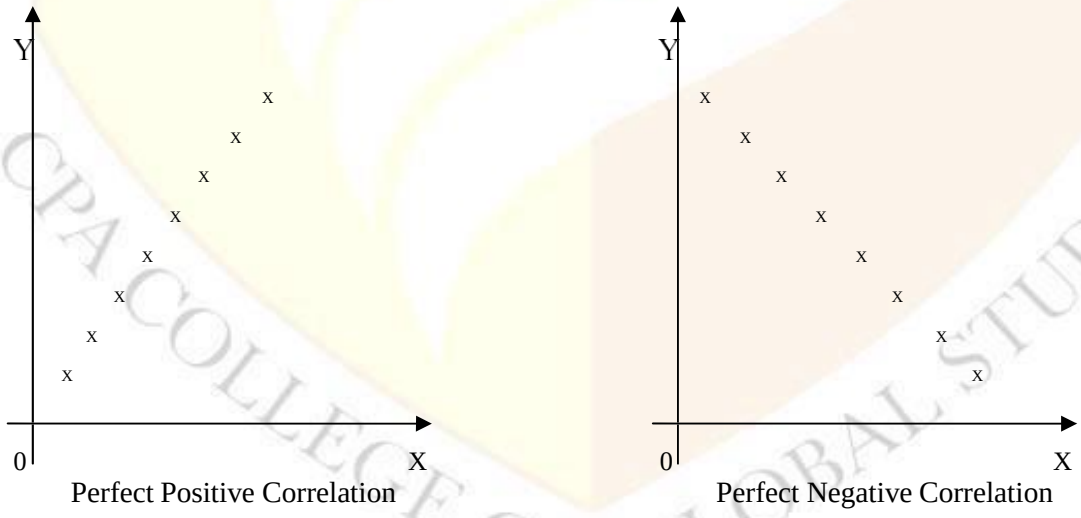
2) Spear man's Rank correlation method

3) Concurrent deviation method

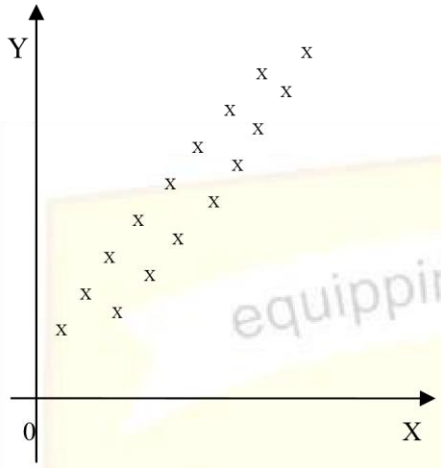
**Scatter Diagram**

This is the simplest method for ascertaining the correlation between variables. Under this method all the values of the two variable are plotted in a chart in the form of dots. Therefore, it is also known as dot chart. By observing the scatter of the vario us dots, we can form an idea that whether the variables are related or not.

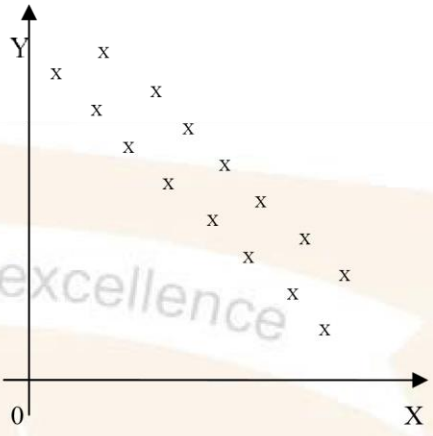
A scatter diagram indicates the direction of correlation and tells us how closely the two variables under study are related. The greater the scatter of the dots, the lower is the relationship



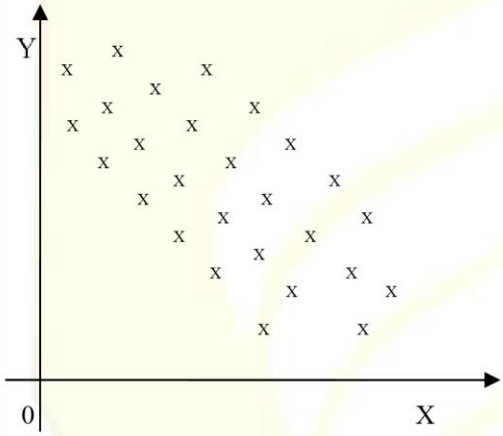

Correlation graph



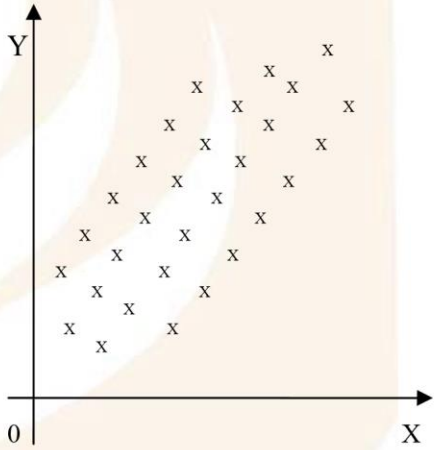

High Degree of Positive Correlation



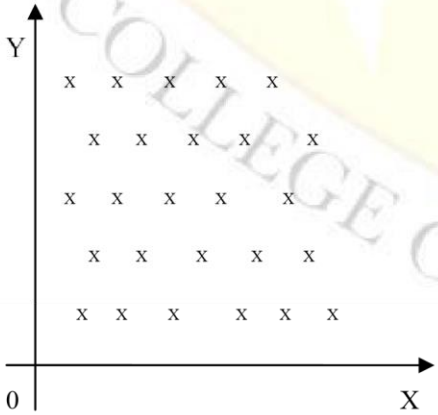

High Degree of Negative Correlation




Low Degree of Positive Correlation




Low Degree of Negative Correlation




No Correlation (r = 0)

**Merits of Scatter Diagram method**

1. It is a simple method of studying correlation between variables.
2. It is a non-mathematical method of studying correlation between the variables. It does not require any mathematical calculations.
3. It is very easy to understand. It gives an idea about the correlation between variables even to a layman.
4. It is not influenced by the size of extreme items.
5. Making a scatter diagram is, usually, the first step in investigating the relationship between two variables.

**Demerits of Scatter diagram method**

1. It gives only a rough idea about the correlation between variables.
2. The numerical measurement of correlation co-efficient cannot be calculated under this method. 3. It is not possible to establish the exact degree of relationship between the variables.

**Correlation graph Method**

Under correlation graph method the individual values of the two variables are plotted on a graph paper. Then dots relating to these variables are joined separately so as to get two curves. By examining the direction and closeness of the two curves, we can infer whether the variables are related or not. If both the curves are moving in the same direction( either upward or downward) correlation is said to be positive. If the curves are moving in the opposite directions, correlation is said to be negative.

**Merits of Correlation Graph Method**

1. This is a simple method of studying relationship between the variable
2. This does not require mathematical calculations.
3. This method is very easy to understand

**Demerits of correlation graph method**:

1. A numerical value of correlation cannot be calculated.
2. It is only a pictorial presentation of the relationship between variables.
3. It is not possible to establish the exact degree of relationship between the variables.

**Karl Pearson's Co-efficient of Correlation**

Karl Pearson's Coefficient of Correlation is the most popular method among the algebraic methods for measuring correlation. This method was developed by Prof. Karl Pearson in 1896. It is also called product moment correlation coefficient.

$$r = \frac{\sum (s-\bar{s})(y-\bar{y})}{f\sum (x-\bar{x})^2 \sum (y-\bar{y})}$$

**Interpretation of Co-efficient of Correlation**

Pearson's Co-efficient of correlation always lies between +1 and -1. The following general rules will help to interpret the Co-efficient of correlation:

1. When r - +1, It means there is perfect positive relationship between variables.
2. When r = -1, it means there is perfect negative relationship between variables.
3. When r = 0, it means there is no relationship between the variables.
4. When _r' is closer to +1, it means there is high degree of positive correlation between variables.
5. When _r' is closer to – 1, it means there is high degree of negative correlation between variables. 6. When _r' is closer to _O', it means there is less relationship between variables.

**Properties of Pearson's Co-efficient of Correlation**

1. If there is correlation between variables, the Co-efficient of correlation lies between +1 and -1.
2. If there is no correlation, the coefficient of correlation is denoted by zero (ie r=0)
3. It measures the degree and direction of change
4. If simply measures the correlation and does not help to predict cansation. 5. It is the geometric mean of two regressions co-efficient.

i.e $r = \sqrt{b_{sy} \cdot b_{yx}}$



## Probable Error and Coefficient of Correlation

Probable error (PE) of the Co-efficient of correlation is a statistical device which measures the reliability and dependability of the value of co-efficient of correlation.

$$PE = 0.6745 \times \frac{1-r^2}{\sqrt{n}}$$

If the value of coefficient of correlation (r) is less than the PE, then there is no evidence of correlation.

If the value of _r' is more than 6 times of PE, the correlation is certain and significant.

By adding and submitting PE from coefficient of correlation, we can find out the upper and lower limits within which the population coefficient of correlation may be expected to lie.

**Uses of PE**:

1) PE is used to determine the limits within which the population coefficient of correlation may be expected to lie.
2) It can be used to test whether the value of correlation coefficient of a sample is significant with that of the population

If r = 0.6 and N = 64, find out the PE and SE of the correlation coefficient. Also determine the limits of population correlation coefficient.

**Merits of Pearson's Coefficient of Correlation:-**

1. This is the most widely used algebraic method to measure coefficient of correlation.
2. It gives a numerical value to express the relationship between variables
3. It gives both direction and degree of relationship between variables
4. It can be used for further algebraic treatment such as coefficient of determination coefficient of non-determination etc.
5. It gives a single figure to explain the accurate degree of correlation between two variables

**Demerits of Pearson's Coefficient of correlation**

1. It is very difficult to compute the value of coefficient of correlation.

2. It is very difficult to understand

**Spearman's Rank Correlation Method**

Pearson's coefficient of correlation method is applicable when variables are measured in quantitative form. But there were many cases where measurement is not possible because of the qualitative nature of the variable. For example, we cannot measure the beauty, morality, intelligence, honesty etc in quantitative terms. However it is possible to rank these qualitative characteristics in some order.

$$(R) = 1 - \frac{6XD2}{N3—N}$$



**Merits of Rank Correlation method**

1. Rank correlation coefficient is only an approximate measure as the actual values are not used for calculations
2. It is very simple to understand the method.
3. It can be applied to any type of data, ie quantitative and qualitative
4. It is the only way of studying correlation between qualitative data such as honesty, beauty etc.
5. As the sum of rank differences of the two qualitative data is always equal to zero, this method facilitates a cross check on the calculation.

**Demerits of Rank Correlation method**

1. Rank correlation coefficient is only an approximate measure as the actual values are not used for calculations.
2. It is not convenient when number of pairs (ie. N) is large
3. Further algebraic treatment is not possible.
4. Combined correlation coefficient of different series cannot be obtained as in the case of mean and standard deviation. In case of mean and standard deviation, it is possible to compute combine arithmetic mean standard deviation.

**Concurrent Deviation Method**:

Concurrent deviation method is a very simple method of measuring correlation. Under this method, we consider only the directions of deviations. The magnitudes of the values are completely ignored. Therefore, this method is useful when we are interested in studying correlation between two variables in a casual manner and not interested in degree (or precision).

Under this method, the nature of correlation is known from the direction of deviation in the values of variables. If deviations of 2 variables are concurrent, then they move in the same direction, otherwise in the opposite direction.

The formula for computing the coefficient of concurrent deviation is: -

$$r = \pm\sqrt{\pm\frac{(2c-N)}{N}}$$

Where N = No. of pairs of symbol

C = No. of concurrent deviations (ie, No. of + signs in _dx dy‘ column)

**Steps:**

1 Every value of _X‘ series is compared with its proceeding value. Increase is shown by _+‘ symbol and decrease is shown by _-_

2 The above step is repeated for _Y‘ series and we get _dy‘

3 Multiply _dx‘ by _dy‘ and the product is shown in the next column. The column heading is _dxdy‘.

4 Take the total number of _+‘ signs in _dxdy‘ column. _+‘ signs in _dxdy‘ column denotes the concurrent deviations, and it is indicated by _C‘.

5 Apply the formula:

$$r = \pm\sqrt{\pm\left(\frac{2c-N}{N}\right)}$$

If $2c \geq N$, then r = +ve and if $2c < N$, then r = −ve.

# REGRESSION ANALYSIS

## Introduction:-

Correlation analysis analyses whether two variables are correlated or not. After having established the fact that two variables are closely related, we may be interested in estimating the value of one variable, given the value of another. Hence, regression analysis means to analyses the average relationship between two variables and thereby provides a mechanism for estimation or predication or forecasting.

The term ‗Regression‘ was firstly used by Sir Francis Galton in 1877. The dictionary meaning of the term ‗regression‘ is ―stepping back‖ to the average.

## Definition:

―Regression is the measure of the average relationship between two or more variables in terms of the original units of the date‖.

―Regression analysis is an attempt to establish the nature of the relationship between variables-that is to study the functional relationship between the variables and thereby provides a mechanism for prediction or forecasting‖.

## Types of Regression:-

There are two types of regression. They are linear regression and multiple regressions.

## Linear Regression:

It is a type of regression which uses one independent variable to explain and/or predict the dependent variable.

## Multiple Regression:

It is a type of regression which uses two or more independent variable to explain and/or predict the dependent variable.

## Regression Lines:

Regression line is a graphic technique to show the functional relationship between the two variables X and Y. It is a line which shows the average relationship between two variables X and Y.

If there is perfect positive correlation between 2 variables, then the two regression lines are winding each other and to give one line. There would be two regression lines when there is no

perfect correlation between two variables. The nearer the two regression lines to each other, the higher is the degree of correlation and the farther the regression lines from each other, the lesser is the degree of correlation.

**Properties of Regression lines**:-

1. The two regression lines cut each other at the point of average of X and average of Y ( i.e $\bar{X}$ and $\bar{Y}$)
2. When r = 1, the two regression lines coincide each other and give one line.

3. When r = 0, the two regression lines are mutually perpendicular. **Regression Equations (Estimating Equations)**

Regression equations are algebraic expressions of the regression lines. Since there are two regression lines, therefore two regression equations. They are :-

1. Regression Equation of X on Y:- This is used to describe the variations in the values of X for given changes in Y.
2. Regression Equation of Y on X :- This is used to describe the variations in the value of Y for given changes in X.

**Regression Equation of Y on X:-**

$$Y = a + bx$$

The normal equations to compute _a' and _b' are: -

$$\Sigma y = Na + b\Sigma x$$

$$\Sigma xy = a\Sigma x + b\Sigma x^2$$

**Regression Equation of X on Y:-**

$$X = a + by$$

The normal equations to compute _a' and _b' are:-

$$\Sigma x = Na + n\Sigma y$$

$$\Sigma xy = a\Sigma y + b\Sigma y^2$$

**Properties of Regression Coefficient:**

1. There are two regression coefficients. They are $b_{xy}$ and $b_{yx}$
2. Both the regression coefficients must have the same signs. If one is +ve, the other will also be a +ve value.
3. The geometric mean of regression coefficients will be the coefficient of correlation. $r = \sqrt{b_{sy}.b_{ys}}$.
4. If x $\tilde{x}$ and $\tilde{y}$ are the same, then the regression coefficient and correlation coefficient will be the same.

**Computation of Regression Co-efficient**

Regression co-efficient can be calculated in 3 different ways:

1. Actual mean method

Regression coefficient x on y ( $b_{xy}$) $= \frac{\Sigma sy}{\Sigma y^2}$

Regression coefficient y on x ($b_{yx}$) = $\frac{\Sigma sy}{\Sigma s^2}$

| **Correlation** | **Regression** |
|---|---|
| It studies degree of relationship between variables | It studies the nature of relationship between variables |
| It is not used for prediction purposes | It is basically used for prediction purposes |
| It is basically used as a tool for determining the degree of relationship | It is basically used as a tool for studying cause and effect relationship |
| There may be nonsense correlation between two variables | There is no such nonsense regression |
| There is no question of dependent and independent variables | There must be dependent and independent variables |

# QUANTITATIVE TECHNIQUES FOR BUSINESS

## MODULE 3

**4th semester Bcom cooperation** **Set theory**

### Set

Set theory is the mathematical theory of well-determined collections, called sets

### Null set

In mathematical sets, the null set, also called the empty set, is the set that does not contain anything

### Subset of a set.

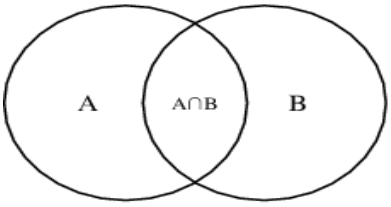
In mathematics, a set A is a subset of a set B if all elements of A are also elements of B; B is then a superset of A.

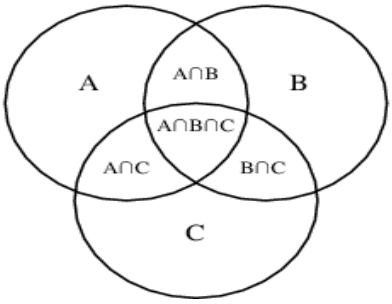
### Disjoint sets

In mathematics, two sets are said to be disjoint sets if they have no element in common.

### Representation sets by venn diagram

A **Venn diagram** is an illustration that uses circles to show the relationships among things or finite groups of things



**Permutation**

A permutation of a set is, loosely speaking, an arrangement of its members into a sequence or linear order, or if the set is already ordered, a rearrangement of its elements.

**Combination**

A combination is a mathematical technique that determines the number of possible arrangements in a collection of items where the order of the selection does not matter

**Example 2: Consider a set having 5 elements a, b, c, d, e. In how many ways 3 elements can be selected (without repetition) out of the total number of elements.**

**Solution:** Given X = {a, b, c, d, e} 3

are to be selected.

Therefore, P53=5! (5−3)!
= 60

**Example 3: It is required to seat 5 men and 4 women in a row so that the women occupy the even places. How many such arrangements are possible?**

**Solution:** We are given that there are 5 men and 4 women. I.e.

there are 9 positions.

The even positions are: 2nd, 4th, 6th and the 8th places

These four places can be occupied by 4 women in P(4, 4) ways = 4! =

4. 3. 2. 1

= 24 ways

The remaining 5 positions can be occupied by 5 men in P (5, 5) = 5!

= 5.4.3.2.1

= 120 ways

Therefore, by the Fundamental Counting Principle,

Total number of ways of seating arrangements = 24 x 120

= 2880

**Question**: Two fair coins are tossed simultaneously. What is the probability of getting only one head?

**Solution**:

When 2 coins are tossed, the possible outcomes can be {HH, TT, HT, and TH}.

Thus, the total number of possible outcomes = 4

Getting only one head includes {HT, TH} outcomes.

So number of desired outcomes = 2

Therefore, probability of getting only one head

=Number off avoidable outcomes Total number of possible outcomes =24=12

**1.** The king, queen and jack of clubs are removed from a deck of 52 playing cards and then shuffled. A card is drawn from the remaining cards. Find the probability of getting: (i) A heart

(ii) A queen

(iii) A club

(IV) '9' of red color

**Solution:**

Total number of card in a deck = 52

Card removed king, queen and jack of clubs

Therefore, remaining cards = 52 - 3=49

Therefore, number of favorable outcomes = 49

**(I)** a heart

Number of hearts in a deck of 52 cards= 13

Therefore, the probability of getting 'a heart'

$$P(A) = \frac{\textit{Number of favorable outcomes}}{\textit{Total number of possible outcome}}$$

= 13/49

**(ii)** A queen

Number of queen = 3

[Since club's queen is already removed]

Therefore, the probability of getting 'a queen t'

$$P(B) = \frac{\textit{Number of favorable outcomes}}{\textit{Total number of possible outcome}}$$

$= 3/49$

**(iii)** a club

Number of clubs in a deck in a deck of 52 cards = 13

According to the question, the king, queen and jack of clubs are removed from a deck of 52 playing cards In this case, total number of clubs = 13 - 3 = 10

Therefore, the probability of getting 'a club'

$$P(C) = \frac{\textit{Number of favorable outcomes}}{\textit{Total number of possible outcome}}$$

$= 10/49$

**(IV)** '9' of red color

Cards of hearts and diamonds are red cards

The card 9 in each suit, hearts and diamonds = 1

Therefore, total number of '9' of red color = 2

Therefore, the probability of getting '9' of red color

$$P(D) = \frac{\textit{Number of favorable outcomes}}{\textit{Total number of possible outcome}}$$

$= 2/49$

# QUANTITATIVE TECHNIQUES FOR BUSINESS

## MODULE 4

## 4th semester Bcom cooperation

**Theoretical Distribution**

## PROBABILITY DISTRIBUTION

A probability distribution is a statistical function that describes all the possible values and likelihoods that a random variable can take within a given range

## BINOMIAL DISTRIBUTION

The binomial distribution is a probability distribution that summarizes the likelihood that a value will take one of two independent values under a given set of parameters or assumptions.

## PROPERTIES:

1. Binomial distribution is applicable when the trials are independent and each trial has just two outcomes success and failure.

2. It is applied in coin tossing experiments, sampling inspection plan, genetic experiments and so on.

3. Binomial distribution is known as bi-parametric distribution as it is characterized by two parameters n and p.

This means that if the values of n and p are known, then the distribution is known completely.

## Binomial Distribution Formula

$$P(x) = \binom{n}{x} p^x q^{n-x} = \frac{n!}{(n-x)!\,x!} p^x q^{n-x}$$

where
$n$ = the number of trials (or the number being sampled)
$x$ = the number of successes desired
$p$ = probability of getting a success in one trial
$q = 1 - p$ = the probability of getting a failure in one trial

**Example**

A set of three similar coins are tossed 100 times with the following results

| Number of heads | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| Frequency | 36 | 40 | 22 | 2 |

Fit a binomial distribution and estimate the expected frequencies.

**Solution**

| $x$ | $f$ | $fx$ |
|---|---|---|
| 0 | 36 | 0 |
| 1 | 40 | 40 |
| 2 | 22 | 44 |
| 3 | 2 | 44 |
| Total | 100 | 90 |

(i) Mean $\bar{x} = \frac{\sum fx}{\sum f} = \frac{90}{100} = 0.9$

*(ii)* $p = \frac{\bar{x}}{n} = \frac{0.9}{3} = 0.3$

*(iii)* $q = 1 - p = 1 - 0.3 = 0.7$

*(iv)* $P(x) = nC_x p^x q^{n-x} = 3C_x \ 0.3_x \ 0.7^{3-x}$

*(v)* $P(0) = 3C_0 \ 0.3^0 \ (0.7)^{3-0} = 0.7^3 = 0.343$

*(vi)* $F(0) = N \times P(0) = 100 \times 0.343 = 34.3$

*(vii)* $F(x+1) = \frac{n-x}{x+1} \times \frac{p}{q} \times F(x)$

$\therefore \quad F(1) = F(0+1) = \frac{3-0}{0+1} \times \frac{0.3}{0.7} \times 34.3 = 44.247$

$F(2) = F(1+1) = \frac{3-1}{1+1} \times \frac{0.3}{0.7} \times 44.247 = 19.03$

$F(3) = F(2+1) = \frac{3-2}{2+1} \times \frac{0.3}{0.7} \times 19.03 = 2.727$

**POISSON DISTRIBUTION**

Poisson distribution is a probability distribution that can be used to show how many times an event is likely to occur within a specified period of time

## Poisson Distribution Formula

$$P(X = x) = \frac{\lambda^x e^{-\lambda}}{x!}$$

where

$x$ = 0, 1, 2, 3, ...

$\lambda$ = mean number of occurrences in the interval

$e$ = Euler's constant $\approx$ 2.71828

**Example:**

The following mistakes per page were observed in a book

| Number of Mistakes (per page) | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| Number of pages | 211 | 90 | 19 | 5 | 0 |

Fit a Poisson distribution and estimate the expected frequencies.

**Solution**

| $x$ | $f$ | $fx$ |
|---|---|---|
| 0 | 211 | 0 |
| 1 | 90 | 90 |
| 2 | 19 | 38 |
| 3 | 5 | 15 |
| 4 | 0 | 0 |
| Total | 325 | 143 |

(i) Mean $\overline{x} = \frac{\Sigma fx}{\Sigma f} = \frac{143}{325} = 0.44$

(ii) $\lambda = \overline{x} = \quad = 0.44$

*(iii)* $P(X = x) = \frac{e^{-\lambda}\lambda^{x}}{x!} = \frac{e^{-0.44}(0.44)^{x}}{x!}$

*(iv)* $P(0) \quad = \frac{e^{0.44} \times 0.44^{0}}{0!} = e^{-0.44} \quad = 0.6440$ (from the Poisson table)

(v) $F(0) \;=\; N \times P(0) \;= 325 \times 0.6440 = 209.43$

*(vi)* $F(x+1) = \frac{\lambda}{x+1} F(x)$

$F(1) \;=\; F(0+1) \;= \frac{0.44}{0+1} \times 209.43 = 92.15$

$F(2) \;=\; F(1+1) \;= \frac{0.44}{1+1} \times 92.15 = 20.27$

$F(3) \;=\; F(2+1) \;= \frac{0.44}{2+1} \times 2.972$

$F(4) \;=\; F(3+1) \;= \frac{0.44}{3+1} \times 2.97 = 0.33$

**NORMAL DISTRIBUTION**

Normal distribution, also known as the Gaussian distribution, is a probability distribution that is symmetric about the mean, showing that data near the mean are more frequent in occurrence than data far from the mean.

**Properties**

- The mean, mode and median are all equal.  The curve is symmetric at the center (i.e. around the mean, μ).
- Exactly half of the values are to the left of center and exactly half the values are to the right.
- The total area under the curve is 1.

Example:

Find expected frequencies for the following data, if its calculated mean and standard deviation are 79.945 and 5.545.

| Class | 60-65 | 65-70 | 70-75 | 75-80 | 80-85 | 85-90 | 90-95 | 95-100 |
|---|---|---|---|---|---|---|---|---|
| Frequency | 3 | 21 | 150 | 335 | 326 | 135 | 26 | 4 |

$$f(x) = \frac{1}{5.545\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-79.945}{5.545}\right)^2}$$

| Class | Lower Class boundry ($X_i$) | $z_i = \frac{X_i - \mu}{\sigma}$ | $\phi(Z_i) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z} e^{-\frac{z^2}{2}} dz$ | $\Delta\phi(Z_i) = \phi(Z_{i+1}) - \phi(Z_i)$ | Expected frequencies $N \Delta \phi(Z_i)$ |
|---|---|---|---|---|---|
| Below 60 | - ∞ | - ∞ | 0 | 0.001 | *1* |
| 60 - 65 | 60 | -3.59693 | 0.001 | 0.0026 | $2.6 \approx 3$ |
| 65 - 70 | 65 | -2.69522 | 0.0036 | 0.0331 | $33.1 \approx 33$ |
| 70 - 75 | 70 | -1.79351 | 0.0367 | 0.15 | 150 |
| 75 - 80 | 75 | -0.89179 | 0.1867 | 0.3173 | $317.3 \approx 317$ |
| 80 - 85 | 80 | 0.009919 | 0.504 | 0.3146 | $314.6 \approx 315$ |
| 85 - 90 | 85 | 0.911632 | 0.8186 | 0.1463 | $146.3 \approx 146$ |
| 90 - 95 | 90 | 1.813345 | 0.9649 | 0.0318 | $31.8 \approx 32$ |
| 95 – 100 | 95 | 2.715059 | 0.9967 | 0.0032 | $3.2 \approx 3$ |
| 100 and Over | 100 | 3.616772 | 0.9999 | | |

**QUANTITATIVE TECHNIQUES FOR BUSINESS**

**MODULE 5**
**4th semester Bcom cooperation**
**Quantitative approach to decision making**

## MODELING

Models are representation of reality. A model may be defined as an idealized representation of a real life system

## PROPERTIES OF A GOOD MODEL

1. Simple
2. It should contain very few variable
3. A model should not take much time in its construction

## ADVANTAGES OF A MODEL

1. It describes problems more concisely
2. It indicates limitation and scope of the problem
3. It facilitates dealing with the problem
4. Analyze the problem

## TYPES OF MODEL

1. Iconic models
2. Analogue models
3. Symbolic models

## PROGRAMMING MODELS

1. Allocation models
2. Sequencing models
3. Queuing models
4. Inventory models
5. Decision models
6. Network models
7. Simulation models
8. Search models
9. Replacement models

## DECISION THEORY

It is a process of choosing an alternative course pof action when a number of alternatives exist

## PROCESS

1. Perceiving the need for decision making
2. Determining the objectives
3. Collection of relevant information

4. Evaluating alternative course of action
5. Choosing the best alternative

## COMPONENTS OF A DECISION PROBLEM

1. Decision maker
2. Number alternatives
3. Different states of nature
4. Outcome of the decision

## TYPES OF DECISION MAKING SITUATION

1. Decision making under certainty
2. Decision making under uncertainty
   Maximax criterion
   Minimax criterion
   Maximin criterion
   Laplace criterion
3. Decision making under risk

## PAYOFF TABLE

A profit table can be a useful way to represent and analyze a scenario where there is a range of possible outcomes and a variety of possible responses. A payoff table simply illustrates all possible profits/losses and as such is often used in decision making under uncertainty

## DECISION TREE

A decision tree is a decision support tool that uses a tree-like model of decisions and their possible consequences, including chance event outcomes, resource costs, and utility. It is one way to display an algorithm that only contains conditional control statements